

Institut de Mathématiques de Toulouse  
118, route de Narbonne  
31062 Toulouse cedex 9  
France

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**A report on the PhD thesis of Hanna Ćmiel entitled “Continuity of Roots and Values for Valued Fields”.**

The main object of study of the thesis is a valued field  $(K, v)$  as well as the polynomial ring  $K[x]$  in one variable, endowed with the Gauß valuation, also denoted by  $v$ . We fix an extension of  $v$  to the algebraic closure  $\tilde{K}$  of  $K$  and, again, denote this extension by  $v$ . Two elements  $\alpha, \beta \in \tilde{K}$  are said to be **close** to each other if the value  $v(\alpha - \beta)$  is large, in the sense that is made precise in each particular instance. Similarly, polynomials  $f, g \in K[x]$  are said to be close to each other if the value  $v(f - g)$  is large.

The main question addressed in the thesis of Hanna Ćmiel is the following.

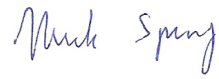
Consider two polynomials  $f, g \in K[x]$ . Can we guarantee, by taking  $f$  and  $g$  sufficiently close to each other, that their roots are also pairwise close? This question is answered in a complete, precise and quantitative manner.

Several different answers are given, depending on the precise hypotheses imposed on  $f$  and  $g$  and the precise form of the conclusion that one is looking for.

The main results of the thesis are Theorems 2.1.1, 2.1.3, 2.1.8, 2.2.2, 2.3.2, 4.1.5, 4.2.2, 4.2.4, 4.2.5 and 4.3.1. In §5 Theorem 4.2.2 is extended to an analogous statement on continuity of roots and poles of rational functions (Theorem 5.2.1). In §6 two results are proved about continuity of factorization into irreducible factors in the henselization of the given valued field  $(K, v)$  (Theorems 6.1.3 and 6.2.5). I found all of these results to be very interesting and non-trivial from the technical point of view. The exposition is extremely precise and clear.

To summarize, the thesis of H. Ćmiel constitutes a significant contribution to the study of valued fields. I feel strongly that the submitted thesis fulfills the conditions set by Article 187 ust. 1-3 ustawy z dnia 20 lipca 2018 r. Prawo

o szkolnictwie wyższym i nauce (Dz. U. z 2022 r. poz. 574 ze zm.).



Mark Spivakovsky  
Directeur de Recherche, CNRS  
Institut de Mathématiques de  
Toulouse  
Université Paul Sabatier