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Report on the thesis entitled "Continuity of roots and values for valued fields" by Hanna Ćmiel

The main topic of the thesis is the following problem. For a valued field (K, v) we consider an extension of v to the algebraic closure $\tilde{K}$ of K, which we call again v. If we take take two polynomials $f, g \in K[x]$, then one can ask the following. If the corresponding coefficients of f and g are "close", then can we conclude that the roots of f and g are "close"?

A more precise formulation of the above question is the following. Assume that $f = a_0 + \ldots + a_n x^n$ and $g = b_0 + b_1 x + \ldots + b_m x^m$ (for simplicity, we complete with zeros, if necessary, to have the same index n). Suppose that $\alpha_1, \ldots, \alpha_r \in \tilde{K}$ are all the roots of f and $\beta_1, \ldots, \beta_s \in \tilde{K}$ are all the roots of g. For ϵ in the valued group, if $v(a_i - b_i) > \epsilon$ for every i, $1 \leq i \leq n$, then can we find a pairing of the roots α_i's and β_j's such that the value of their difference is larger than ϵ? It is important to observe that for valued fields, two elements are thought to be "close" if the value of their difference is large.

The question mentioned above is very important and has been extensively studied. This thesis presents various improvements of the known results. In order to present such improvements, the author developed many important tools.

In Chapter 1, it is presented the basic results and objects of the thesis. Chapter 2 is devoted to improve some of the known results. The main approach used in this chapter is to obtain results using finite induction on the degrees of the polynomials.

In Chapter 3, the theory of Newton Polygons is presented. These are very important objects in valuation theory. The author develops various technical tools that allow to present more precise results on continuity of roots in Chapter 4.

In Chapter 5, the previous results are generalized for zeros and poles of rational functions (instead of zeros of polynomials). Finally, in Chapter 6, the author presents applications of the previous results to other problems. On particular and very interesting of these applications is in ramification theory of valuations. One example is formula (6.11), which is a generalization of the famous Fundamental Inequality (in this case, an equality). This formula is very interesting because it gives a precise statement on how the *defect* of each extension contributes to the degree of the extension.

The author has shown broad knowledge of her field of studies. Moreover, the results in this thesis have strong potential for applications in problems of valuation theory and related areas. I empathise that this thesis is very detailed and well-written. Because of this, I recommend to accept it.

The submitted thesis fulfills the conditions set by Article 187 ust. 1-3 ustawy z dnia 20 lipca 2018 r. Prawo o szkolnictwie wyższym i nauce (Dz. U. z 2022 r. poz. 574 ze zm.)