



WYDZIAŁ MATEMATYCZNO – FIZYCZNY
Instytut Matematyki

Zaprasza na wykład pod tytułem:

**FROM THE NEWTON ALGORITHM OVER
HENSEL'S LEMMA TO A FRAMEWORK
FOR FIXED POINT THEOREMS**

który wygłosi:

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For real functions, the Newton Algorithm is a nice tool to approximate zeros. An analogue works in valued fields, such as the fields of p -adic numbers. It can be used to prove Hensel's Lemma, which states the existence of p -adic roots of suitable polynomials. Hensel's Lemma also holds in other valued fields (such as power series fields), which can be much larger than the p -adics. Still, it can be proved using the Newton algorithm, but the algorithm may not deliver the root after the first ω many iterations; transfinite induction is then needed. For algebraists who do not like transfinite induction, Sibylla Priess-Crampe developed in 1990 an elegant alternative. The setting of the Newton algorithm gives rise to a contracting function whose unique fixed point, if it exists, is a root of the polynomial. Starting with her ultrametric version of Banach's Fixed Point Theorem, Priess then proved, together with Paulo Ribenboim, a large number of fixed point theorems and other results for valued fields and ultrametric spaces.

My own contribution to these developments was to introduce a slightly different point of view: the study of the properties of polynomials as functions on valued fields. This led to what Priess and Ribenboim called an "attractor theorem", and resulted in a new understanding of Hensel's Lemma. The point is now to show the surjectivity of the polynomial function on a suitable part of the valued field. If that part contains 0, surjectivity yields a root of the polynomial.

The attractor theorem is a powerful tool to prove multi-dimensional versions of Hensel's Lemma in various settings, and even an infinite-dimensional Implicit Function Theorem. The latter is used by Bernard Teissier in his approach to local uniformization, the local form of resolution of singularities (which is still a deep open problem in positive characteristic).

Since 2011 I have developed, in joint work with my wife Katarzyna Kuhlmann, a general framework for fixed point theorems that work with contracting functions. We drew our inspiration from the theory of ultrametric spaces. The general framework (which we call "ball spaces") does not only apply to ultrametric spaces, but also to metric and topological spaces, lattices and posets, ordered fields and abelian groups, and more applications that still remain to be explored. It also allows to shift concepts and results from one application to the other. In joint work with Saharon Shelah, this general approach led to the classification of symmetrically complete ordered fields and abelian groups (in which every descending chain of closed intervals has a nonempty intersection). The further study of ball spaces is expanding and attracts more and more collaborators, such as W. Kubiś (Prague) and R. Bartsch (TU Darmstadt).

Wykład odbędzie się **12 maja 2016 r.** (czwartek)
o godz. **16.00** w sali 212
w budynku Wydziału Matematyczno – Fizycznego.